AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Question Set 5

1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. **[2]**

2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

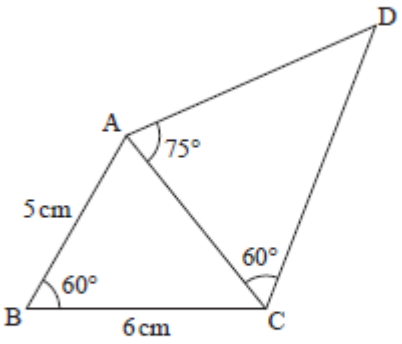

Fig. 2

Calculate the exact value of the length AD. **[4]**

3 Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i} + 7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i} - 12\mathbf{j}$.

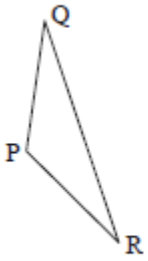

Fig. 3

(a) Show that the triangle PQR is isosceles. **[3]**

(b) Find the position vector of S. **[2]**

4 Fig. 4.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve with x-coordinate $1 + h$.

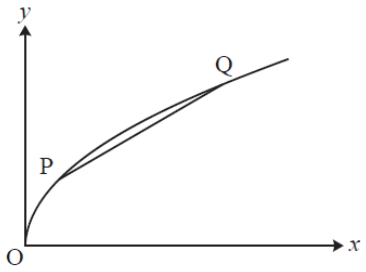

Fig. 4.1

Table 4.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 4.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 4.2 below. Give your answers correct to 6 decimal places where necessary. **[1]**

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1				
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

(b) Explain how the sequence of values in the last column of Table 4.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

(c) Use calculus to find the gradient of the curve at the point P. **[2]**

5 **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

6 The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. **[7]**

7 **In this question you must show detailed reasoning.**

(a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. **[1]**

(b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. **[3]**

(c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. **[5]**

8 **In this question you must show detailed reasoning.**

Fig. 8 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

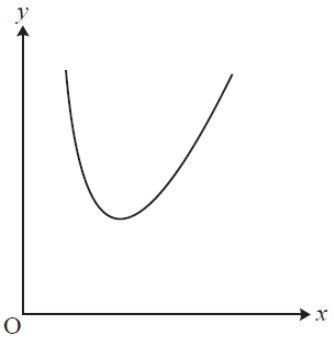

Fig. 8

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

(a) Find the exact area of the triangle OAB. **[6]**

(b) Use calculus to prove that the complete curve has two minimum points and no maximum point. **[6]**

Total Marks for Question Set 5: 49 marks